The Eight Field Way

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Motivation

- We study the compactification of the 6d rank 1 E-string theory to 4d on Riemann surfaces with fluxes under its global symmetry.
- We determine the 4d field theories for many such compactifications, and perform a variety of consistency checks.
- Can be used to better understand 4d field theory.
- Leads to various dynamical predictions for 4d theories: dualities, symmetry enhancement, etc…

Outline

1. Introduction

   - Compactification of 6d $\mathcal{N}$= (1,0) SCFTs

   - E-string SCFT

2. Compactification of the E-string SCFT

3. 4d field theory results

4. Conclusions

Compactification of 6d SCFTs

- We consider compactification of a $\mathcal{N}= (1,0)$ 6d SCFT on a Riemann surface to 4d.
- The type of 4d theory we get depends on the choice of 6d SCFT, Riemann surface and properties of the compactification.
- The 6d construction implies a variety of properties expected from the 4d theory.
- Different ways to construct the same compactification should correspond to dual 4d theories.

Dimensional reduction with fluxes

- Consider the 6d $(1,0)$ SCFT compactified on a Riemann surface.
- Problem: the curvature of the Riemann surface breaks SUSY.
- In order to preserve SUSY we must perform a twist $SO(2)_S \rightarrow SO(2)_S - U(1)_R$ for $U(1)_R$ the Cartan of $SU(2)_R$.
- The twist makes some of the spinors covariantly constant and preserves $\mathcal{N}$=1 SUSY in 4d.
- This breaks $SU(2)_R$ to $U(1)_R$.

Fluxes and flux quantization

- If the 6d SCFT has a global symmetry G then we can turn on flux on the Riemann surface for abelian subgroups of G. For torus compactifications, this breaks SUSY down to $\mathcal{N}$=1 in 4d.
- For each choice of flux we expect a different 4d theory.
- This breaks G to the subgroup that commute with the chosen $U(1)$'s.
- The flux must be quantized: $$\int_{\mathcal{C}_g} F = \frac{2\pi n}{q_{min}}$$
- Flux quantization depends on the global structure of the groups.
- Because of these global issues flux quantization can be quite subtle: two inconsistent fluxes may together form a consistent flux.

Conformal manifold

- Besides fluxes can also turn on flat connections. These correspond to holonomies and preserve only $\mathcal{N}$=1 SUSY. Map to marginal deformations in 4d.
- For a genus $g > 1$ Riemann surface with no punctures, we can count the dimension of the conformal manifold to be:

$$dim\mathcal{M}_g = 3(g-1) + (g-1)dimG_{max} + L$$

- The first term are the marginal deformations corresponding to the complex structure moduli of the Riemann surface.
- The second term are marginal deformations corresponding to holonomies. Here G_{max} is the global symmetry preserved by the flux, and L is the number of abelian components in it.

Anomalies

- Can calculate the 4d t' Hooft anomalies by integrating the 6d anomaly polynomial eight form:

$$\int_{\mathcal{C}_g} I_8(\mathcal{F}) = I_6^{(g,\mathcal{F})}$$

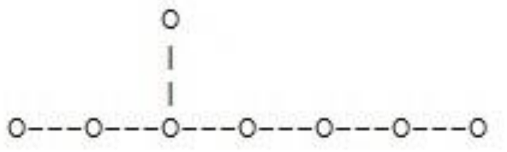
- This requires evaluating the 6d anomaly polynomial.
- Anomaly calculation sensitive to some details of the compactification such as flux.
- Can only calculate anomalies for symmetries inherited from 6d.

Rank 1 E-string theory

- 6d SCFT made of 1 tensor multiplet with an E_8 global symmetry.
- Described in string theory by 1 M5-brane near an M9-plane [Ganor, Hanany, 1996].
- When reduced to 5d, has an effective IR description as an $SU(2) + 8F$ gauge theory [Ganor, Morrison, Seiberg, 1997].
- When reduced to 4d with no flux, gives the rank 1 MN E_8 theory.

Fluxes in E_8

- The global symmetry here is E_8. In compactification without flux this is the expected global symmetry.
- We can consider compactification with flux. We have many different choices preserving different subgroups of E_8.
- For instance, with flux to just one $U(1)$:



$\boldsymbol{U(1) \times E_7}$	$\boldsymbol{U(1) \times SO(14)}$	$\boldsymbol{U(1) \times SU(2) \times E_6}$	$\boldsymbol{U(1) \times SU(8)}$
$\boldsymbol{U(1) \times SU(3) \times SO(10)}$	$\boldsymbol{U(1) \times SU(2) \times SU(7)}$	$\boldsymbol{U(1) \times SU(4) \times SU(5)}$	$\boldsymbol{U(1) \times SU(2) \times SU(3) \times SU(5)}$

Anomalies

- The 6d theory anomaly polynomial was evaluated in [Ohmori, Shimizu, Tachikawa, 2014]. Integrating it we can predict the anomalies of the 4d theories.
- Example for the case of $g = 1$ and flux z preserving $U(1) \times E_7$:

$$a = 2|z|, c = \frac{5}{2}|z|$$

- In this case anomalies are surprisingly simple.
- For other $U(1)$'s: $a = 2|z|\sqrt{\xi}, c = \frac{5}{2}|z|\sqrt{\xi}$, where:

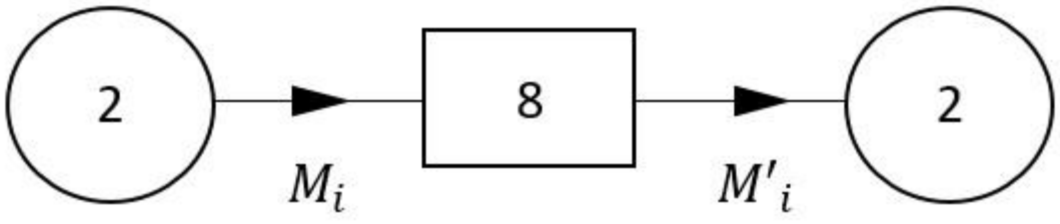
$\boldsymbol{G = E_7, \xi = 1}$	$\boldsymbol{G = SO(14), \xi = 2}$	$\boldsymbol{G = SU(2) \times E_6, \xi = 3}$	$\boldsymbol{G = SU(8), \xi = 4}$
$\boldsymbol{G = SU(3) \times SO(10), \xi = 6}$	$\boldsymbol{G = SU(2) \times SU(7), \xi = 7}$	$\boldsymbol{G = SU(4) \times SU(5), \xi = 10}$	$\boldsymbol{G = SU(2) \times SU(3) \times SU(5), \xi = 15}$

4d field theories

- We next want to determine the corresponding 4d theories.
- Generically, this is the hardest part in the study of compactification of 6d SCFTs.
- We need some stepping stone or guiding principle.
- We have seen that the anomalies for a torus compactification with flux preserving $U(1) \times E_7$ are surprisingly simple.
- We know of a 4d theory that is thought to have E_7 symmetry somewhere on its conformal manifold.

E_7 surprize

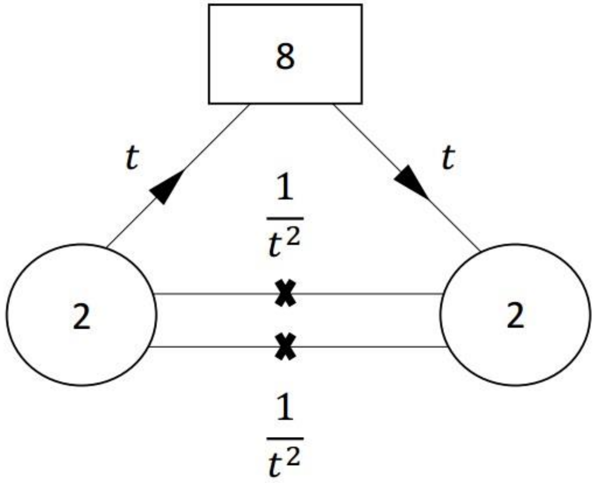
- In [Dimofte, Gaiotto, 2012] a theory was constructed whose spectrum forms characters of E_7.
- Natural question: could this be an E-string compactification.
- Answer: no, since anomalies don’t match.
- But could a generalization of this work?



$$W = M_i^2 M'^2_i$$

E_7 theory

- Indeed a generalization of the theory works.
- Particularly we consider the following field theory:



- We claim that this theory is an E-string compactification on a torus with flux -1 preserving $U(1) \times E_7$.

E_7 theory

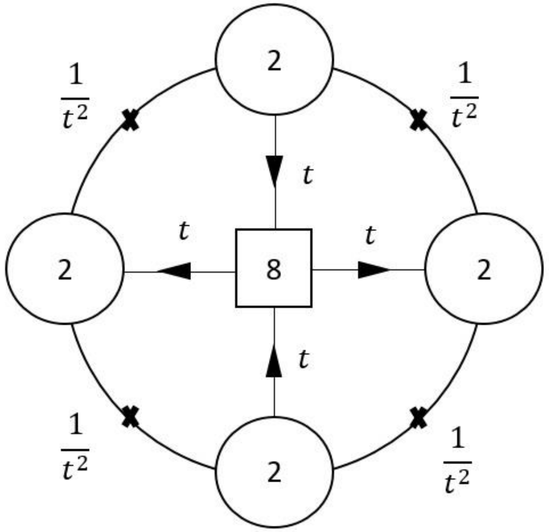
- We can perform various consistency checks.
- Anomalies match.
- The index forms characters of E_7 (no singlets):

$$I_{E_7} = 1 + (pq)^{\frac{2}{3}}(\frac{3}{t^4} + t^2\chi[\mathbf{56}]) - 2pq + (pq)^{\frac{2}{3}}(p+q)(\frac{2}{t^4} + t^2\chi[\mathbf{56}])$$
$$+ (pq)^{\frac{4}{3}}(\frac{6}{t^8} + \frac{1}{t^2}\chi[\mathbf{56}] + t^4(\chi[\mathbf{1463}] - \chi[\mathbf{133}] - 1)) + \dots$$

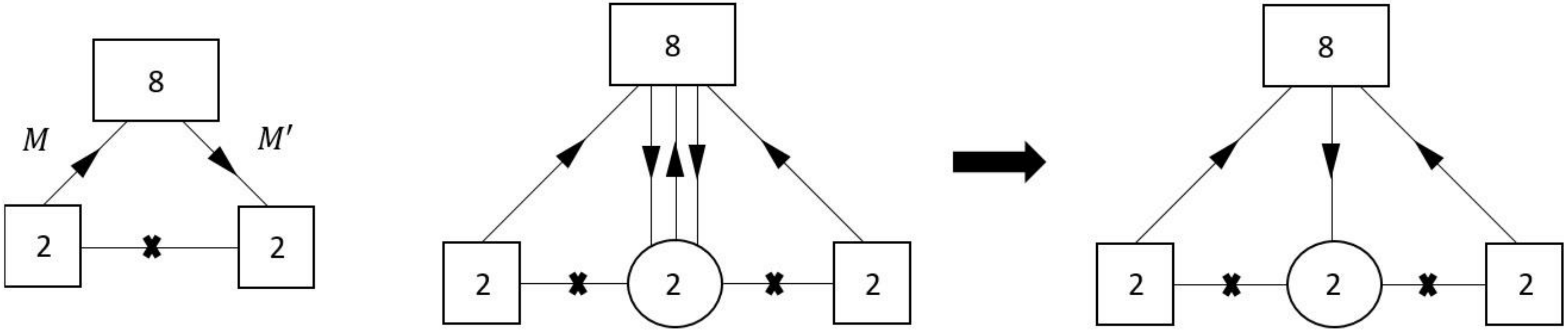
- An E_7 perserving deformation of this theory yields the surprise.

E_7 theories with more flux

- Can generalize to cases with more flux.
- With flux $-z$ this will be a circle of $2z$ $SU(2)$ gauge groups connected to a single $SU(8)$ global symmetry group via a cubic superpotential.
- We find: anomalies match, index forms characters of E_7 ...
- $z = 2$ example:



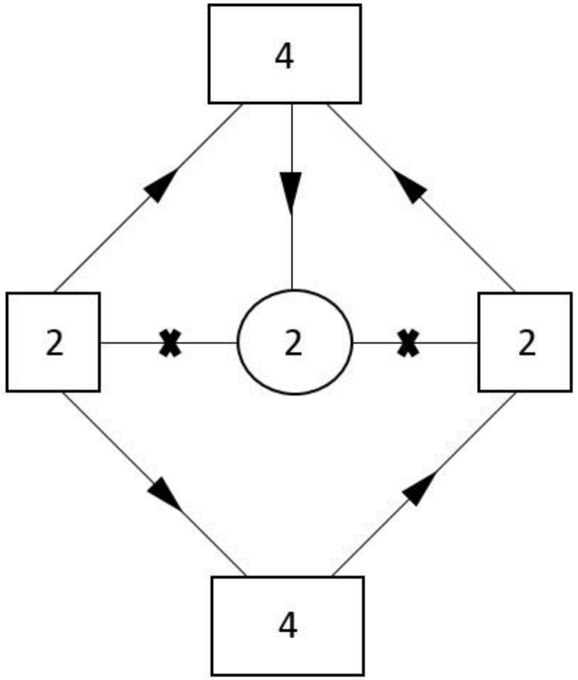
E_7 tube



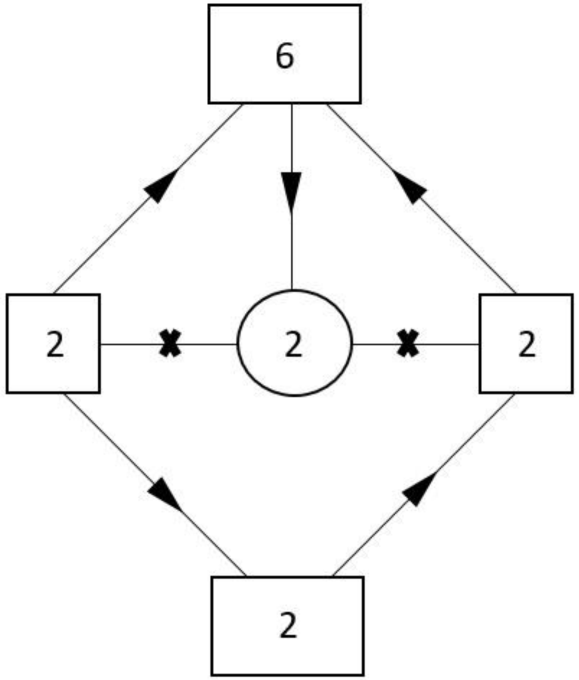
- Can decompose the torus theories to theories which we associate with tube compactification.
- The tubes are spheres with two punctures and flux $-\frac{1}{2}$ preserving $U(1) \times E_7$.
- To each puncture we associate an $SU(2)$ global symmetry.
- Connecting two punctures together is done by identifying their $SU(2)$ global symmetries, gauge them, and add 8 fundamental chirals Φ coupled to M trough a superpotential $W = \Phi M$.

Other tubes

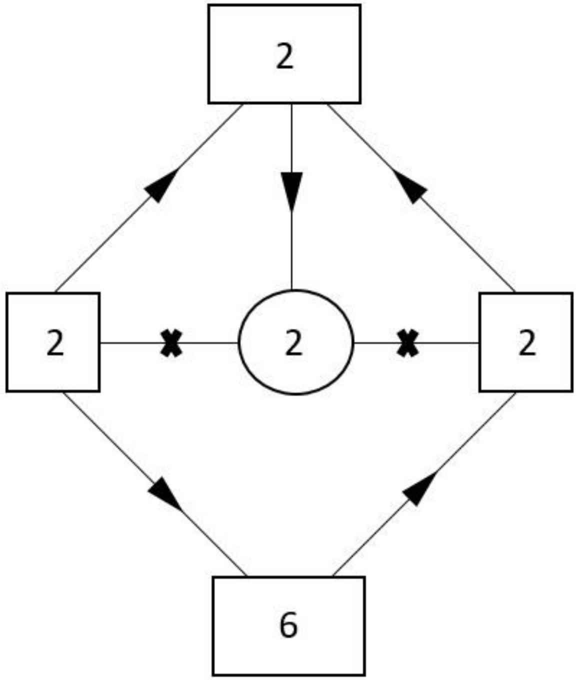
- We can find tubes for other embeddings of $U(1) \subset E_8$.



$U(1) \times SO(14)$



$U(1) \times SU(2) \times E_6$



$U(1) \times E'_7$

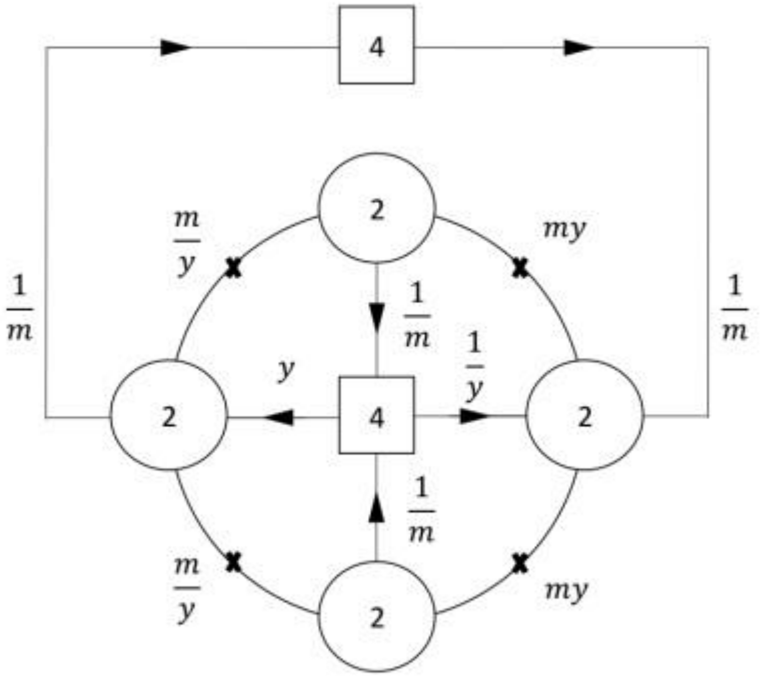
$U(1) \times SO(14)$ theory

- Can create field theory corresponding to an E-string compactification on a torus with flux -1 preserving $U(1) \times SO(14)$.
- Anomalies match the 6d computation.
- Index forms characters of $SO(14)$.

$$\mathcal{I} = 1 + \frac{2}{m^2}\chi[\mathbf{14}](pq)^{\frac{1}{2}} + \frac{1}{m}\chi[\mathbf{64}](pq)^{\frac{3}{4}} + \frac{2}{m^2}\chi[\mathbf{14}](pq)^{\frac{1}{2}}(p+q)$$
$$+ pq(m^4 + \frac{1}{m^4}(3\chi[\mathbf{104}] + \chi[\mathbf{91}] - 1)) + \dots$$

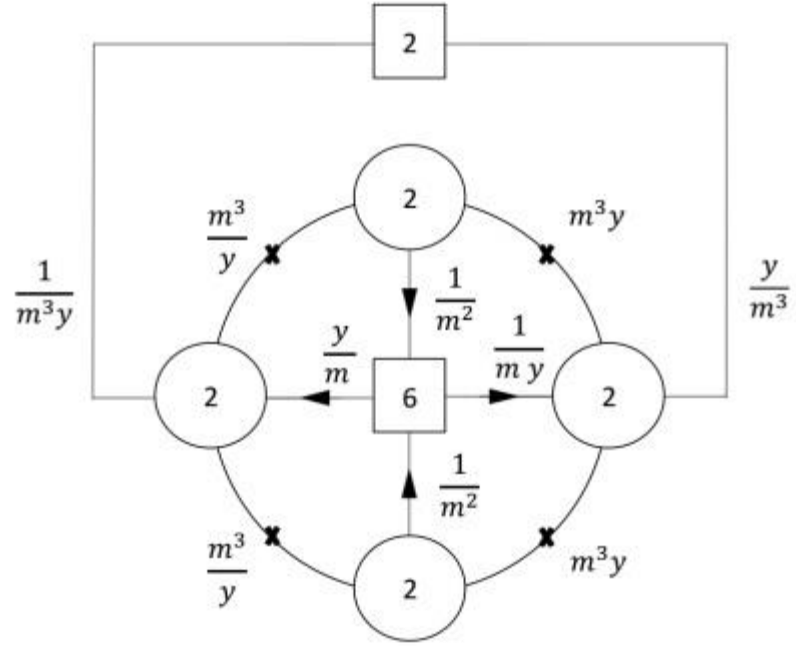
$$\chi[\mathbf{14}] = y^2 + \frac{1}{y^2} + \chi[\mathbf{6},\mathbf{1}] + \chi[\mathbf{1},\mathbf{6}]$$

$$\chi[\mathbf{64}] = y(\chi[\mathbf{4},\overline{\mathbf{4}}] + \chi[\overline{\mathbf{4}},\mathbf{4}]) + \frac{1}{y}(\chi[\mathbf{4},\mathbf{4}] + \chi[\overline{\mathbf{4}},\overline{\mathbf{4}}])$$



$U(1) \times SU(2) \times E_6$ theory

- Can create field theory corresponding to an E-string compactification on a torus with flux -1 preserving $U(1) \times SU(2) \times E_6$.
- Anomalies match the 6d computation.
- Index forms characters of $SU(2) \times E_6$.



$$\mathcal{I} = 1 + \frac{3}{m^6}\chi[\mathbf{2},\mathbf{1}](pq)^{\frac{1}{3}} + \frac{2}{m^4}\chi[\mathbf{1},\overline{\mathbf{27}}](pq)^{\frac{5}{9}} + \frac{3}{m^6}\chi[\mathbf{2},\mathbf{1}](pq)^{\frac{1}{3}}(p+q)$$
$$+ \frac{3}{m^{12}}(1+2\chi[\mathbf{3},\mathbf{1}])(pq)^{\frac{2}{3}} + \frac{1}{m^2}\chi[\mathbf{2},\mathbf{27}](pq)^{\frac{7}{9}} + \frac{6}{m^{10}}\chi[\mathbf{2},\overline{\mathbf{27}}](pq)^{\frac{8}{9}}$$
$$+ pq\frac{2}{m^{18}}(4\chi[\mathbf{4},\mathbf{1}] + 3\chi[\mathbf{2},\mathbf{1}]) + \ldots$$

$$\chi[\mathbf{2},\mathbf{1}] = y^2 + \frac{1}{y^2}, \quad \chi[\mathbf{1},\overline{\mathbf{27}}] = \chi[\mathbf{2},\mathbf{6}]_{SU(2)\times SU(6)} + \chi[\mathbf{1},\overline{\mathbf{15}}]_{SU(2)\times SU(6)}$$

Other tubes: consistency checks

- Armed with the four tubes we presented, we can now perform a variety of consistency checks.
- Particularly, we can consider connecting different tubes together to form torus compactifications with fluxes for several $U(1)$`s.
- We again have predictions for the global symmetry, anomalies, etc…, and these can be checked against the field theory results.
- In all cases we checked, the field theory results agree with the 6d predictions.

Trinion

- Consider constructing a genus g Riemann surface by gluing together a collection of three punctured spheres (trinion).
- From 6d: know the anomalies of the of the resulting 4d theory.
- We also know the gluing procedures.
- Therefore we can determine the anomalies of the trinion.
- Particularly, the ones involving the 6d R-symmetry are independent of the flux, and are given by: $Tr(R) = -10, Tr(R^3) = 2$.
- Can we find theories with these anomalies?

T_A and T_e

- Fortunately we have an infinite class of models that have these anomalies.
- Trinions associated with the compactification of the 6d SCFT living on 2 M5-branes probing a C^2/Z_2 singularity happen to have the same anomalies.
- Perhaps one of them is an E-string trinion.
- One model seems promising: T_A. It is a non-Lagrangian theory that is encountered when studying dualities of these class of models.

T_A and T_e

- The interesting thing about T_A is its global symmetry: $SU(2)^3 \times SO(8) \times U(1)^2$. The $SU(2)^3$ can be naturally associated with the three punctures, and the $SO(8)$ with the $SU(8)$ that appears in the 4d tubes.
- This is further supported by the index:

$$\mathcal{I} = 1 + (\frac{2}{a^4 t^2} + \frac{a^4}{t^2} + t(\mathbf{2}_A \mathbf{8}_v + \mathbf{2}_C \mathbf{8}_c + \mathbf{2}_B \mathbf{8}_s) + t a^4 \mathbf{2}_A \mathbf{2}_C \mathbf{2}_B + \frac{1}{a^4} \mathbf{28}) pq$$
$$+ (-\mathbf{28} - \mathbf{3}_A - \mathbf{3}_C - \mathbf{3}_B - 1 - 1) pq + \cdots$$

- From anomalies and symmetry considerations: we find that it is actually a mass deformation of the T_A that is an E-string trinion. We call this theory T_e.

Combining to form theories with E_8

- We can now consider constructing a genus g Riemann surface from an equal number of T_e's and its complex conjugate.
- The complex conjugate T_e corresponds to a compactification with opposite flux.
- Therefore the resulting surface has zero flux and we expect an E_8 global symmetry.
- Indeed index forms characters of E_8: $\mathcal{I} = 1 + (\mathbf{248}(g-1) + 3g - 3)qp + \cdots$

$$\mathbf{248} = 1 + \frac{1}{t^4} + t^4 + (2t^2 + 1 + \frac{2}{t^2})\mathbf{28} + \mathbf{35_V} + \mathbf{35_S} + \mathbf{35_C}$$

- We also note that the dimension of the conformal manifold is as expected.

Combining to form theories with $U(1) \times E_7$

- We can also combine the T_e trinions with themselves to a form a genus g Riemann surface with flux.
- By studying the global symmetry and anomalies of the resulting theories we find that we must associate to T_e a flux of $\frac{3}{4}$ preserving $U(1) \times E_7$.
- By connecting T_e trinions with themselves and with the E_7 tubes we previously introduced: can create a large number of examples.
- In all cases we checked: anomalies, dualities, conformal manifold, etc…, all agree with 6d predictions.

4. Conclusions

- We have constructed the field theories corresponding to a large variety of compactifications of the rank 1 E-string theory on Riemann surfaces with fluxes.
- The 6d construction leads to predictions for the 4d theories, which can be used as consistency checks.
- Our proposal has passed a large number of consistency checks.
- This can be used to understand and predict surprising symmetry enhancements in 4d.

Open questions

- Does this constructions imply interesting new 4d dualities?
- Generalizations to related 6d SCFTs:
    - More tensors- Higher rank E-string.
    - Add vectors and hypers.
    - Combine the above - a vast array of 6d SCFTs related to the rank 1 E-string theory.
- Is there a TQFT structure for the index?
- What about reductions to 3d?

Thank you